Sample Question Paper - 35
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours *Maximum Marks : 40*

General Instructions :

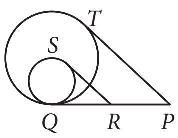
1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Find the mean of the following distribution :

Class	3-5	5-7	7-9	9-11	11-13
Frequency	5	10	10	7	8

2. In the given figure, PQ is the common tangent to both the circles. SR and PT are also tangents. If $SR = 16$ cm, $PT = 28$ cm, then find RP.



3. For what values of k, the quadratic equation $x^2 - 2kx + 5k = 0$ has equal roots?

OR

Solve for x : $2x^2 + 6\sqrt{3}x - 60 = 0$

4. If the numbers $2n + 1$, $3n + 4$ and $6n + 1$ are in A.P., find n and hence find the numbers.

5. Form the quadratic equation from $x = \sqrt{6 + \sqrt{6 + \sqrt{6 +}}}$ ...., where x is a natural number. Also, solve the equation to find the value of x.

6. Two cubes each of volume 27 cm^3 are joined end to end to form a solid. Find the surface area of the resulting cuboid.

OR

A cuboidal solid block of metal 49 cm × 44 cm × 18 cm is melted and formed into a solid sphere. Calculate the radius of the sphere.

SECTION - B

7. Find out the mode for the following data showing frequency with which profits are made:

Profit (in ₹ 10)	3-4	4-5	5-6	6-7	7-8	8-9	9-10
Frequency	83	27	25	50	75	38	18

8. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. Find the distance of the car from the base of the tower.

9. Draw a circle with the help of a round bottle cap. Take a point outside the circle. Construct the pair of tangents from this point to the circle.

OR

Geometrically divide a line segment of length 9.6 cm internally in the ratio of 4 : 1.

10. Find the median of the following data :

Class interval	0-10	10-20	20-30	30-40	40-50	Total
Frequency	8	16	36	34	6	100

SECTION - C

11. Two ships are approaching a light-house from opposite directions. The angles of depression of the two ships from the top of the light-house are 30° and 45°. If the distance between the two ships is 100 m, find the height of the light-house. [Use $\sqrt{3} = 1.732$]

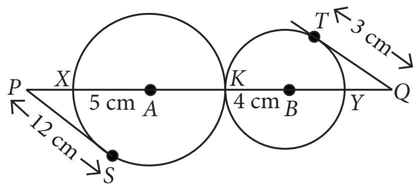
OR

Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles. $\left[\text{Use } \sqrt{3} = 1.732\right]$

12. From a solid cylinder of height 20 cm and diameter 12 cm, a conical cavity of height 8 cm and radius 6 cm is hollowed out. Find the total surface area of the remaining solid. $\left[\text{Use } \pi = \frac{22}{7}\right]$

Case Study - 1

13. In a maths class, the teacher draws two circles that touch each other externally at point K with centres A and B and radii 5 cm and 4 cm respectively as shown in the figure.



Based on the above information, answer the following questions.

(i) Find the value of PA.

(ii) Find the value of PK.

Case Study - 2

14. While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an A.P. If the number on the n^{th} spot is $5n - 1$, then answer the following questions to help the player in spotting the clues.

(i) Which number is on the 24^{th} spot?

(ii) Which spot is numbered as 119?

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. The frequency distribution table from the given data can be drawn as :

Class	Class marks (x_i)	Frequency (f_i)	$f_i x_i$
3-5	4	5	20
5-7	6	10	60
7-9	8	10	80
9-11	10	7	70
11-13	12	8	96
Total		40	326

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{326}{40} = 8.15$

2. Since tangents drawn from an external point to a circle are equal in length.

$\therefore$ $PQ = PT = 28$ cm and $RQ = RS = 16$ cm

Now, $RP = PQ - RQ = (28 - 16)$ cm $= 12$ cm

3. For roots of $x^2 - 2kx + 5k = 0$ to be equal,

Discriminant, $D = 0$

$\therefore$ $(-2k)^2 - 4(1)(5k) = 0 \Rightarrow 4k^2 - 20k = 0$

$\Rightarrow$ $k^2 - 5k = 0 \Rightarrow k(k - 5) = 0 \Rightarrow k = 0$ or $k = 5$

OR

We have, $2x^2 + 6\sqrt{3}x - 60 = 0$

$\Rightarrow$ $x^2 + 3\sqrt{3}x - 30 = 0$

$\Rightarrow$ $x^2 + 5\sqrt{3}x - 2\sqrt{3}x - 30 = 0$

$\Rightarrow$ $x(x + 5\sqrt{3}) - 2\sqrt{3}(x + 5\sqrt{3}) = 0$

$\Rightarrow$ $(x + 5\sqrt{3})(x - 2\sqrt{3}) = 0$

$\Rightarrow$ $x + 5\sqrt{3} = 0$ or $x - 2\sqrt{3} = 0$

$\Rightarrow$ $x = -5\sqrt{3}$ or $x = 2\sqrt{3}$.

4. Let $2n + 1$, $3n + 4$ and $6n + 1$ are in A.P.

$\therefore$ $(3n + 4) - (2n + 1) = (6n + 1) - (3n + 4)$

$\Rightarrow$ $n + 3 = 3n - 3 \Rightarrow 6 = 2n \Rightarrow n = 3$

$\therefore$ Numbers are $2(3) + 1 = 7$, $3(3) + 4 = 13$, $6(3) + 1 = 19$.

5. We have, $x = \sqrt{6 + \sqrt{6 + \sqrt{6 +}}}$ ....

Squaring both sides, we get $x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6}}}$ ....

$\Rightarrow$ $x^2 = 6 + x$

$\Rightarrow$ $x^2 - x - 6 = 0$, which is the required quadratic equation.

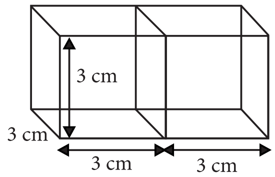
Also, $x^2 + 2x - 3x - 6 = 0$

$\Rightarrow$ $x(x + 2) - 3(x + 2) = 0 \Rightarrow (x + 2)(x - 3) = 0$

$\Rightarrow$ $x = 3, -2$ ($x = -2$ is not possible as x is a natural number)

$\therefore$ $x = 3$

6. Let the edge of each cube be a cm.



$\therefore$ Volume of each cube $= a^3$ cm^3

i.e., $a^3 = 27 = (3)^3 \Rightarrow a = 3$

$\therefore$ Surface area of the cuboid $= 2(lb + bh + hl)$

$= 2(6 \times 3 + 3 \times 3 + 3 \times 6) = 2 \times 45 = 90$ cm^2

OR

Let r be the radius of sphere.

Now, volume of block = volume of sphere

$\Rightarrow$ $49 \times 44 \times 18 = \dfrac{4}{3} \times \dfrac{22}{7} \times r^3$

$\Rightarrow$ $\dfrac{49 \times 44 \times 18 \times 3 \times 7}{4 \times 22} = r^3$

$\Rightarrow$ $(7 \times 3)^3 = r^3$ or $r = 21$ cm

$\therefore$ Radius of sphere = 21 cm

7. From the given data, we observe that the highest frequency is 83, which lies in the class interval 3-4.

$\therefore$ Modal class is 3-4.

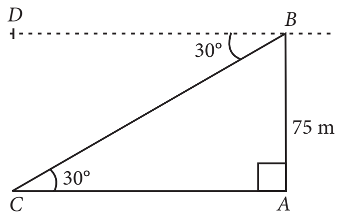
So, $l = 3$, $h = 1$, $f_1 = 83$, $f_0 = 0$, $f_2 = 27$

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 3 + \left(\dfrac{83 - 0}{2 \times 83 - 0 - 27}\right) \times 1 = 3 + \dfrac{83}{166 - 27} = 3 + \dfrac{83}{139}$

$= 3 + 0.5971 = 3.5971$

8.



Let $AB = 75$ m be the height of tower and C be the position of car.

Then, according to question,

In right ΔABC, $\cot 30° = \dfrac{AC}{AB}$

$\Rightarrow$ $AC = AB \cot 30°$

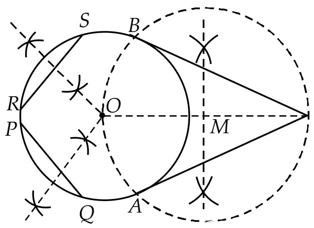
$\Rightarrow \quad AC = 75 \times \sqrt{3} \Rightarrow AC = 75\sqrt{3}$ m

Thus, the distance of the car from the base of the tower is $75\sqrt{3}$ m.

9. Steps of construction :

Step-I : Draw the circle using a round bottle cap.

Step-II : Draw two non parallel chords *PQ* and *RS* on this circle.



Step-III : Draw the perpendicular bisectors of *PQ* and *RS* such that they intersect at *O*.

Therefore, *O* is the centre of the given circle.

Step-IV : Take a point *P* outside this circle.

Step-V : Join *OP* and bisect it. Let *M* be the mid-point of *OP*.

Step-VI : Taking *M* as centre and *OM* as radius, draw a circle intersecting the given circle at *A* and *B*.

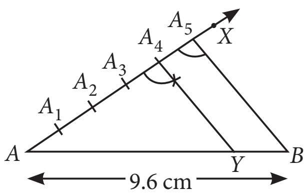
Step-VII : Join *PA* and *PB*.

Thus, *PA* and *PB* are the required two tangents.

OR

Steps of construction :

Step-I : Draw a line segment *AB* = 9.6 cm.



Step-II : Draw a ray *AX* making an acute angle with *AB*.

Step-III : Locate (4 + 1 =) 5 points $A_1, A_2, ..., A_5$ on *AX* such that $AA_1 = A_1A_2 = ... = A_4A_5$.

Step-IV : Join A_5B.

Step-V : Through A_4, draw A_4Y parallel to A_5B meeting *AB* at point *Y* such that $\angle AA_5B = \angle AA_4Y$.

Thus, point *Y* divides *AB* in the ratio of 4 : 1, *i.e.*, $AY : YB = 4 : 1$

10. The frequency distribution table from the given data can be drawn as :

Class interval	Frequency (f_i)	Cumulative frequency (*c.f.*)
0-10	8	8
10-20	16	24
20-30	36	60
30-40	34	94
40-50	6	100
Total	100	

Here, $N = 100$, $\frac{N}{2} = 50$, which lies in the class interval 20-30

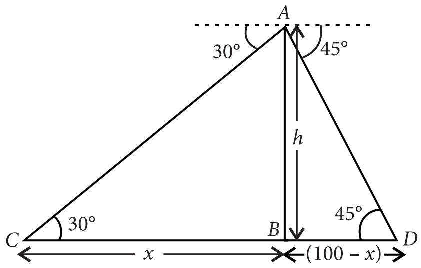
$\therefore$ Median class is 20-30.

$$\text{Median} = l + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 20 + \left[\frac{50 - 24}{36}\right] \times 10 = 20 + 7.22 = 27.22$$

11. Let *AB* be the light house and two ships *C* and *D* are 100 m apart.

Let $BC = x$ then, $BD = 100 - x$



In ΔABC, $\tan 30° = \frac{h}{x} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow \quad h = \frac{x}{\sqrt{3}}$ ...(i)

In ΔABD, $\tan 45° = \frac{h}{100 - x}$

$\Rightarrow \quad 1 = \frac{h}{100 - x} \Rightarrow 100 - x = h$

$\Rightarrow \quad 100 - x = \frac{x}{\sqrt{3}}$ (From (i))

$\Rightarrow \quad 100\sqrt{3} = x(\sqrt{3} + 1) \Rightarrow \quad x = \frac{100(\sqrt{3})}{\sqrt{3} + 1}$

$\therefore \quad x = \frac{100\sqrt{3}}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{300 - 173.2}{2} = 63.4$

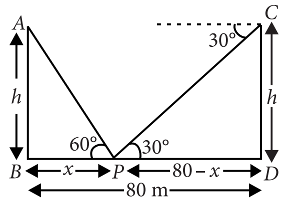
$\therefore \quad h = \frac{63.4}{1.732} = 36.6$ m

OR

Let *AB* and *CD* be two poles of height *h* m.

Let *P* be a point on road such that *BP* = *x* m so that $PD = BD - BP = (80 - x)$m

In ΔABP,



$\frac{h}{x} = \tan 60°$

$\Rightarrow \quad h = \sqrt{3}x$ ...(i)

In ΔCDP,

$\frac{h}{80-x} = \tan 30° \Rightarrow \quad h = \frac{1}{\sqrt{3}}(80-x)$ ...(ii)

From (i) and (ii), we have

$\sqrt{3}x = \frac{1}{\sqrt{3}}(80-x) \quad \Rightarrow \quad 3x = 80 - x$

$\Rightarrow \quad 4x = 80$ or $x = 20$

Distance of point P from AB = 20 m

Distance of point P from CD = 80 – 20 = 60 m

Height of each pole, $h = x\sqrt{3} = 20\sqrt{3}$

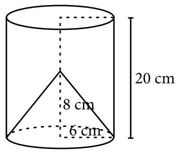
$= 20 \times 1.732 = 34.64$ m

12. Height of the cylinder, $h_1 = 20$ cm

$\therefore$ Radius of the cylinder, $r = \frac{12}{2} = 6$ cm

Height of the cone, $h_2 = 8$ cm

Radius of the cone, $r = 6$ cm



Total surface area of the remaining solid

= Area of the top face of the cylinder + CSA of the cylinder + CSA of the cone.

Slant height of the cone, $l = \sqrt{(8)^2 + (6)^2}$

$= \sqrt{64 + 36} = \sqrt{100} = 10$ cm

CSA of the cone = πrl

$= \frac{22}{7} \times 6 \times 10 = \frac{1320}{7}$ cm^2

CSA of the cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times 6 \times 20 = \frac{5280}{7}$ cm^2

Area of the top face of the cylinder $= \pi r^2$

$= \frac{22}{7} \times (6)^2 = \frac{792}{7}$ cm^2

$\therefore$ Total surface area of the remaining solid

$= \left(\frac{1320}{7} + \frac{5280}{7} + \frac{792}{7}\right)$ cm^2

$= \frac{7392}{7}$ cm^2 = 1056 cm^2

13. Here, AS = 5 cm, BT = 4 cm [$\because$ Radii of circles]

(i) Since, radius at point of contact is perpendicular to tangent.

$\therefore$ By Pythagoras theorem, we have

$PA = \sqrt{PS^2 + AS^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$ cm

(ii) $PK = PA + AK$

$= 13 + 5 = 18$ cm.

14. Number on n^{th} spot = $5n - 1$

i.e., $t_n = 5n - 1$

(i) Number on 24th spot = t_{24}

$= 5(24) - 1 = 119$

(ii) Let n^{th} spot is numbered as 119.

$\therefore$ $t_n = 119$

$\Rightarrow$ $5n - 1 = 119 \Rightarrow 5n = 120 \Rightarrow n = 24$